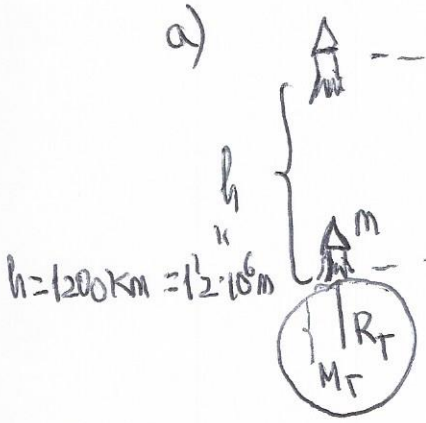
(202) a) En ningún caso nos indica orbitación del cohete.

a)



$h = 1200\,Km = 1'2\cdot10^6\,m$

$R_T = 6370\,Km = 6'37\cdot10^6\,m$

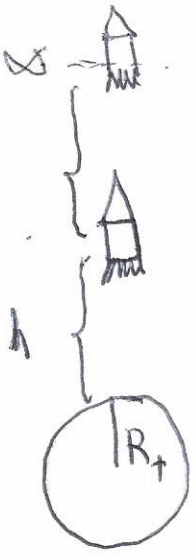
$$E_{Ph} = -G\frac{M_T\cdot m}{(R_T+h)} = -6'67\cdot10^{-11}\cdot\frac{6\cdot10^{24}\cdot600}{(6'37\cdot10^6+1'2\cdot10^6)} = -3'17\cdot10^{10}\,J$$

$$E_{PT} = -G\cdot\frac{M_T\cdot m}{R_T} = -6'67\cdot10^{-11}\cdot\frac{6\cdot10^{24}\cdot600}{6'37\cdot10^6} = -3'77\cdot10^{10}\,J$$

La E_p se habría incrementado y dicho aumento vale:

$$\Delta E_p = E_{Ph} - E_{PT} = -3'17\cdot10^{10}J - (-3'77\cdot10^{10}J) = 6\cdot10^9\,J$$ ha aumentado su E_p.

b) ∞



$E_{P\infty} = 0$, $E_{C\infty} = 0$

$E_{Ph} = -G\frac{M_T\cdot M}{(R_T+h)}$ / $E_{adicional}$

Para escapar a la acción del campo gravitatorio terrestre tendría que situarse a una distancia infinita de la Tierra.

$$E_{Ph} + E_{adicional} = \cancel{E_{P\infty}}^{\,0} + \cancel{E_{C\infty}}^{\,0}$$

$$E_{adicional} = -E_{Ph} \Rightarrow \left(\text{Ya calculada en a)}\right)$$

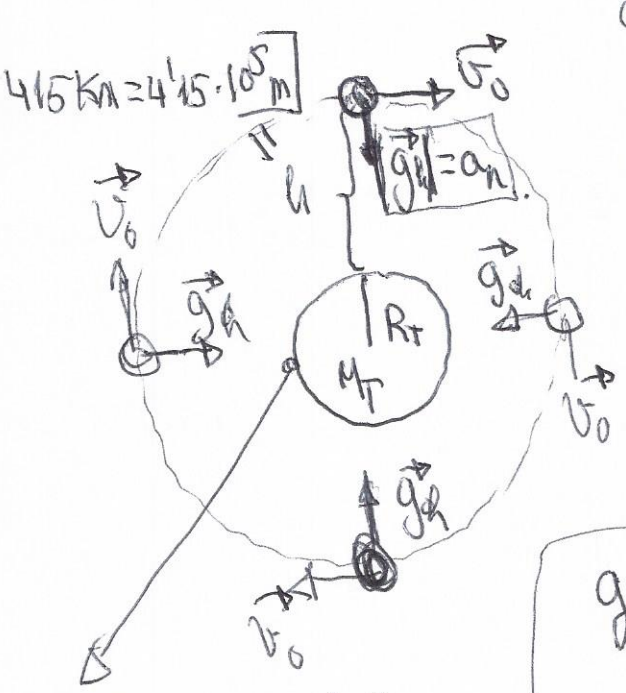
$$E_{adicional} = -\left(-3'17\cdot10^{10}J\right)$$

$E_{adicional} = 3'17\cdot10^{10}J$ tendríamos que suministrarle.

214

a)

Cuando nos pide el valor del campo gravitatorio nos están pidiendo $\vec{g}$, el cual expresaremos solo con el módulo puesto que como vector $\vec{g}$ su dirección sería cambiante en los puntos en donde está un astronauta en su órbita.



$$g_h = \frac{G \cdot M_T}{(R_T+h)^2} \Rightarrow g_h = \frac{g_T \cdot R_T^2}{(R_T+h)^2}$$

Nos dan $g_T = 9'8 \frac{N}{s^2}$ (superficie terrestre)

$$g_T = \frac{G M_T}{R_T^2} \longrightarrow G M_T = g_T \cdot R_T^2$$

Sustituyo el producto

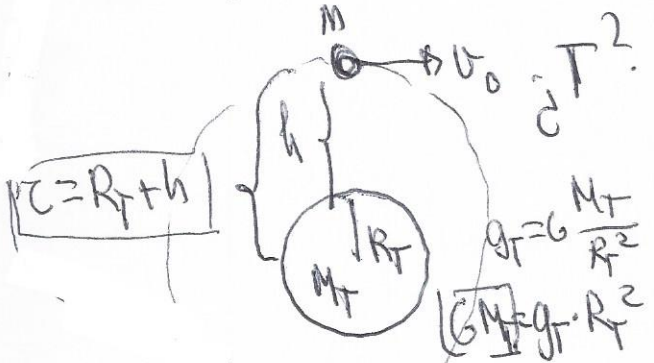
$$g_h = g_T \cdot \frac{R_T^2}{(R_T+h)^2}$$

$$g_h = g_T \left(\frac{R_T}{(R_T+h)}\right)^2$$

$$g_h = 9'8 \cdot \left(\frac{6'37 \cdot 10^6}{6'37 \cdot 10^6 + 4'15 \cdot 10^5}\right)^2 = 8'64 \ m/s^2$$

b) Utilizamos la condición de orbitación

$$F_g = F_n \qquad \text{MCU} \quad v_0 = \frac{2\pi(R_T+h)}{T}$$



$C = R_T + h$

$g_T = G \frac{M_T}{R_T^2}$

$G M_T = g_T \cdot R_T^2$

$$G \frac{M_T \cdot m}{(R_T+h)^2} = m \cdot \frac{v_0^2}{(R_T+h)}$$

$$G \frac{M_T}{(R_T+h)} = \frac{4\pi^2 (R_T+h)^2}{T^2}$$

$$G M_T T^2 = 4\pi^2 (R_T+h)^3$$

$$T = \sqrt{\frac{4\pi^2 (R_T+h)^3}{G \cdot M_T}} = \sqrt{\frac{4\pi^2 (R_T+h)^3}{g_T R_T^2}} = \sqrt{\frac{4\pi^2 (6'37 \cdot 10^6 + 4'15 \cdot 10^5)^3}{9'8 \cdot (6'37 \cdot 10^6)^2}} = 5559'185 \ s$$

SATÉLITE LEO

1'54 h

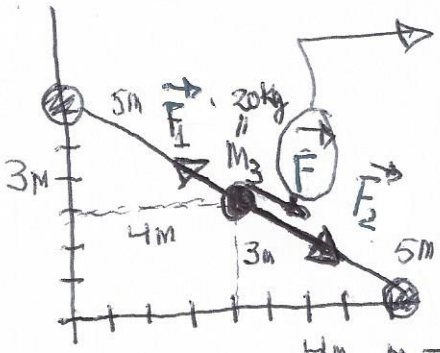
221 a)

Principio de superposición

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$m_1 = 50\,kg$

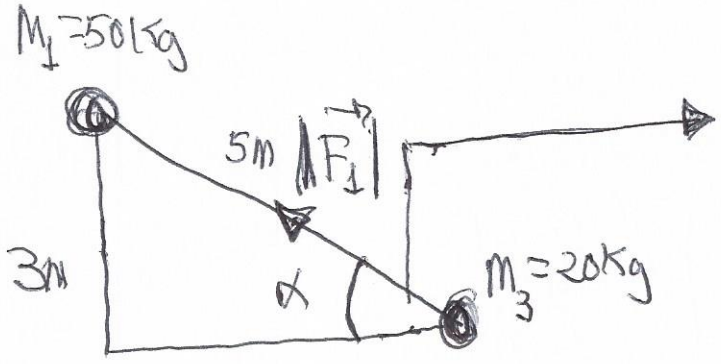
A (0,6)m



$m_2 = 100\,kg$

B (8,0)m.

Primero calculo $\vec{F}_1$



Halla el ángulo α mediante razones trigonométricas.

$$\text{sen}\,\alpha = \frac{\text{cat. opuesto}}{\text{hipotenusa}}$$

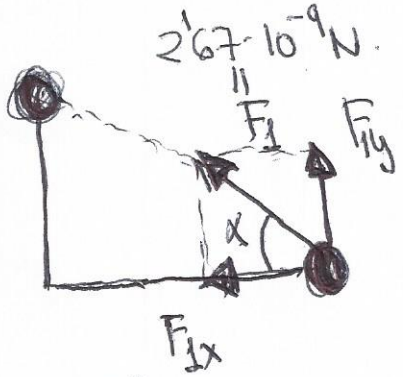
Halla $|\vec{F}_1|$ para después descomponer.

$$\text{sen}\,\alpha = \frac{3}{5} \Rightarrow \alpha = \text{arcsen}\,\frac{3}{5}$$

$$\boxed{\alpha = 36'87^\circ}$$

$$|\vec{F}_1| = G\frac{M_1 \cdot m_3}{r^2} = 6'67 \cdot 10^{-11} \cdot \frac{50 \cdot 20}{5^2} = 2'67 \cdot 10^{-9}\,N$$

↓ lo descompongo.



$$\text{sen}\,\alpha = \frac{F_{1y}}{F_1} \Rightarrow F_{1y} = F_1 \cdot \text{sen}\,\alpha = 2'67 \cdot 10^{-9} \cdot \text{sen}\,36'87^\circ$$

$$F_{1y} = 2'67 \cdot 10^{-9} \cdot \frac{3}{5} = 1'6 \cdot 10^{-9}\,N$$

$$\vec{F}_{1y} = +1'6 \cdot 10^{-9}\,\vec{j}\,(N)$$

$$\cos\alpha = \frac{F_{1x}}{F_1} \Rightarrow F_{1x} = F_1 \cdot \cos\alpha = 2'67 \cdot 10^{-9} \cdot \cos 36'87^\circ$$

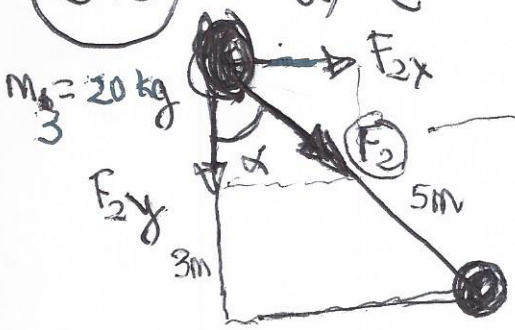
$$F_{1x} = 2'67 \cdot 10^{-9} \cdot \frac{4}{5} = 2'14 \cdot 10^{-9}\,N$$

$$\vec{F}_{1x} = -2'14 \cdot 10^{-9}\,\vec{i}\,(N)$$

$$\vec{F}_1 = \vec{F}_{1x} + \vec{F}_{1y}$$

$$\vec{F}_1 = -2'14 \cdot 10^{-9}\,\vec{i} + 1'6 \cdot 10^{-9}\,\vec{j}\,(N)$$

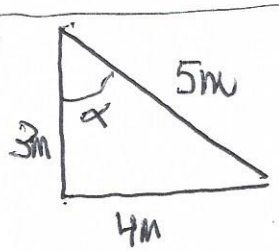
221 a) (continuación) Calculo ahora $\vec{F}_2$



$$F_2 = G\cdot\frac{m_2\cdot m_3}{r^2}$$

$$F_2 = 6'67\cdot10^{-11}\cdot\frac{100\cdot20}{5^2}$$

$$\boxed{F_2 = 5'34\cdot10^{-9}\,N}$$



$$\text{sen}\,\alpha = \frac{\text{cat. opuesto}}{\text{hipotenusa}}$$

$$\text{sen}\,\alpha = \frac{4}{5}$$

$$\alpha = \text{arcsen}\frac{4}{5} = 53'13°$$

$$\text{sen}\,53'13° = \frac{F_{2x}}{F_2} \Rightarrow F_{2x} = F_2\cdot\text{sen}\,53'13° = 5'34\cdot10^{-9}\cdot\frac{4}{5}$$

$$F_{2x} = 4'27\cdot10^{-9}\,N$$

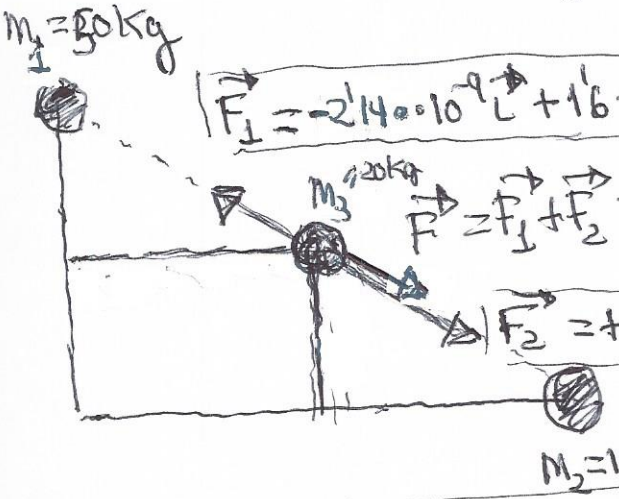
$$\vec{F}_{2x} = +4'27\cdot10^{-9}\,\vec{i}\;(N)$$

$$\cos 53'13° = \frac{F_{2y}}{F_2} \Rightarrow F_{2y} = F_2\cdot\cos 53'13° = 5'34\cdot10^{-9}\cdot\frac{3}{5}$$

$$F_{2y} = 3'2\cdot10^{-9}\,N$$

$$\vec{F}_{2y} = -3'2\cdot10^{-9}\,\vec{j}\;(N)$$

$$\vec{F}_2 = \vec{F}_{2x} + \vec{F}_{2y} \Rightarrow \boxed{\vec{F}_2 = +4'27\cdot10^{-9}\,\vec{i} - 3'2\cdot10^{-9}\,\vec{j}\;(N)}$$



$$\boxed{\vec{F}_1 = -2'14\cdot10^{-9}\,\vec{i} + 1'6\cdot10^{-9}\,\vec{j}\;(N)}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = -2'14\cdot10^{-9}\,\vec{i} + 1'6\cdot10^{-9}\,\vec{j} + 4'27\cdot10^{-9}\,\vec{i} - 3'2\cdot10^{-9}\,\vec{j}$$

$$\boxed{\vec{F}_2 = +4'27\cdot10^{-9}\,\vec{i} - 3'2\cdot10^{-9}\,\vec{j}\;(N)}$$

$$\boxed{\vec{F} = +2'13\cdot10^{-9}\,\vec{i} - 1'6\cdot10^{-9}\,\vec{j}\;(N)}$$

ANÁLISIS DEL RESULTADO: como $\vec{F}_1$ y $\vec{F}_2$ están en la misma dirección, al restar sus módulos, obtendremos el módulo de $\vec{F}$

$|\vec{F}_1| = F_1 = 2'67\cdot10^{-9}\,N$ (ya calculados)

$|\vec{F}_2| = F_2 = 5'34\cdot10^{-9}\,N$

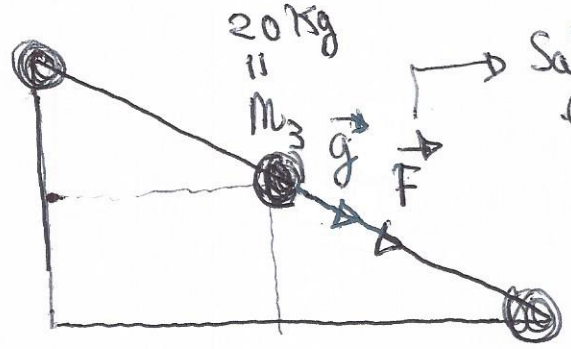
$$|\vec{F}| = \sqrt{x^2 + y^2}$$

$$|\vec{F}| = \sqrt{(2'13\cdot10^{-9})^2 + (-1'6\cdot10^{-9})^2}$$

$$\boxed{|\vec{F}| = 2'66\cdot10^{-9}\,N}$$

$$|\vec{F}| = |\vec{F}_2| - |\vec{F}_1| = 5'34\cdot10^{-9}\,N - 2'67\cdot10^{-9}\,N = \boxed{2'68\cdot10^{-9}\,N}$$

(221) a) Valor del campo gravitatorio en el punto.



→ Sabiendo $\vec{F}$ puedo calcular $\vec{g}$ en el pto haciendo

$$\vec{g} = \frac{\vec{F}}{m_3} = \frac{+2'13\cdot10^{-9}\vec{i} - 1'6\cdot10^{-9}\vec{j}}{20} \quad (N/kg)$$

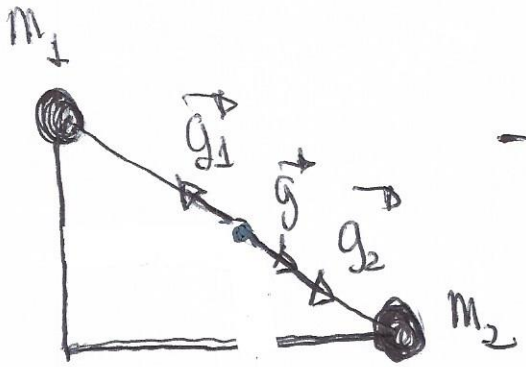
$\vec{F} = +2'13\cdot10^{-9}\vec{i} - 1'6\cdot10^{-9}\vec{j}$ (N)

$$\boxed{\vec{g} = 1'065\cdot10^{-10}\vec{i} - 8\cdot10^{-11}\vec{j}} \quad (N/kg = m/s^2)$$

$$|\vec{g}| = \sqrt{x^2 + y^2}$$

$$|\vec{g}| = \sqrt{(1'065\cdot10^{-10})^2 + (-8\cdot10^{-11})^2}$$

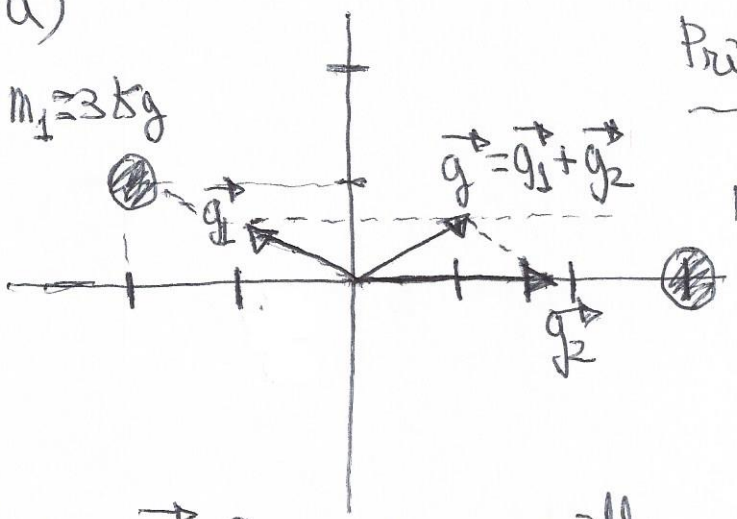
$$\boxed{|\vec{g}| = 1'33\cdot10^{-10}\ N/s^2} \quad (N/kg \text{ sería igual})$$



→ Se podría haber hecho, en caso de no saber $\vec{F}$, aplicando el principio de superposición.

$$\vec{g} = \vec{g}_1 + \vec{g}_2$$

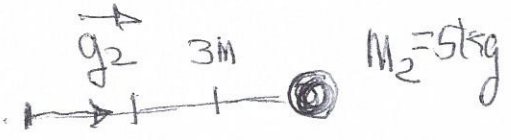
(22) a)



Principio de superposición

$m_2 = 5kg$.

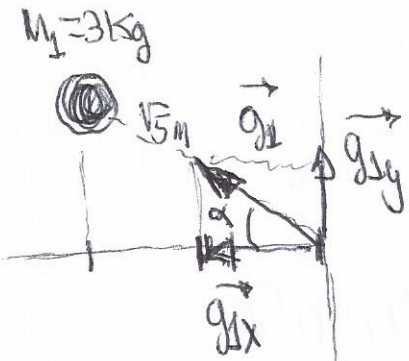
Calculamos $\vec{g_2}$ que es mas sencilla.



$$|\vec{g_2}| = G\frac{M_2}{r_2^2} = 6'67\cdot10^{-11}\cdot\frac{5}{3^2} \simeq 3'7\cdot10^{-11}\ \frac{N}{S^2}$$

$$\vec{g_2} = +3'7\cdot10^{-11}\vec{i}\ \left(\frac{N}{S^2}\right)$$

Para calcular $\vec{g_1}$ la descompongo.



$$|\vec{g_1}| = G\cdot\frac{M_1}{r_1^2}$$

$$|\vec{g_1}| = 6'67\cdot10^{-11}\cdot\frac{3}{(\sqrt{5})^2}$$

$$|\vec{g_1}| = 4\cdot10^{-11}\ N/S^2$$

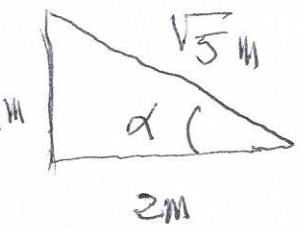
$$\operatorname{sen}\alpha = \frac{g_{1y}}{g_1}$$

$$g_{1y} = g_1\cdot\operatorname{sen}26'56°$$

$$g_{1y} = 4\cdot10^{-11}\cdot\frac{1}{\sqrt{5}}$$

$$g_{1y} = 1'78\cdot10^{-11}\ N/S^2$$

$$\vec{g_{1y}} = 1'78\cdot10^{-11}\vec{j}\ (N/S^2)$$



$$\operatorname{sen}\alpha = \frac{1}{\sqrt{5}} \qquad \alpha = \operatorname{arcsen}\frac{1}{\sqrt{5}}$$

$$\alpha = 26'56°.$$

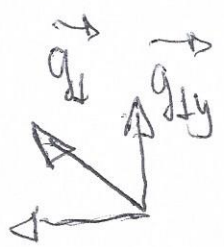
$$\cos\alpha = \frac{g_{1x}}{g_1}$$

$$g_{1x} = g_1\cdot\cos26'56°$$

$$g_{1x} = 4\cdot10^{-11}\cdot\frac{2}{\sqrt{5}}$$

$$g_{1x} = 3'57\cdot10^{-11}$$

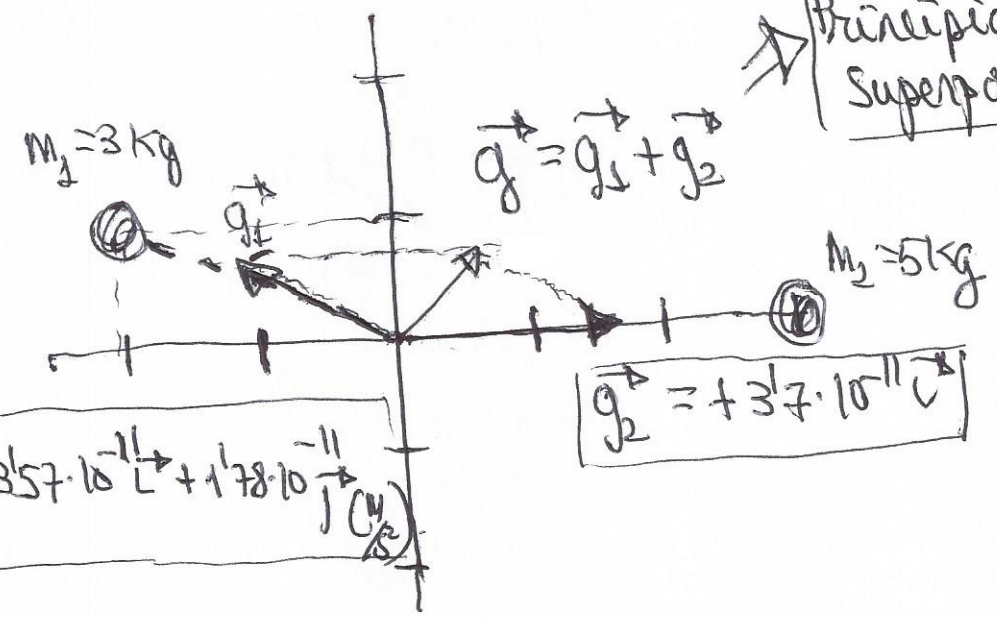
$$\vec{g_{1x}} = -3'57\cdot10^{-11}\vec{i}\ (N/S^2)$$



$$\vec{g_1} = \vec{g_{1x}} + \vec{g_{1y}}$$

$$\vec{g_1} = -3'57\cdot10^{-11}\vec{i} + 1'78\cdot10^{-11}\vec{j}\ \left(\frac{N}{S^2}\right)$$

(222) a) Continuación



⇒ Principio de Superposición

$$\vec{g} = \vec{g_1} + \vec{g_2}$$

$$\vec{g} = -3'57 \cdot 10^{-11}\vec{i} + 1'78 \cdot 10^{-11}\vec{j} + 3'7 \cdot 10^{-11}\vec{i}$$

$$\boxed{\vec{g} = 1'3 \cdot 10^{-12}\vec{i} + 1'78 \cdot 10^{-11}\vec{j}\ (N/kg)}$$

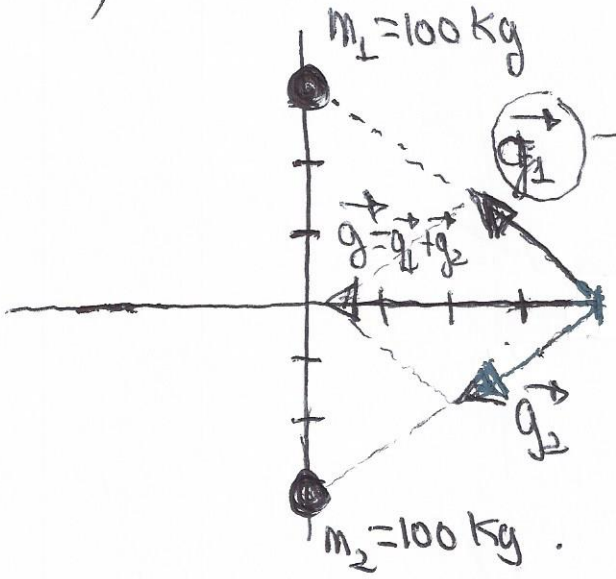
$$|\vec{g}| = \sqrt{g_x^2 + g_y^2}$$

$$|\vec{g}| = \sqrt{(1'3 \cdot 10^{-12})^2 + (1'78 \cdot 10^{-11})^2}$$

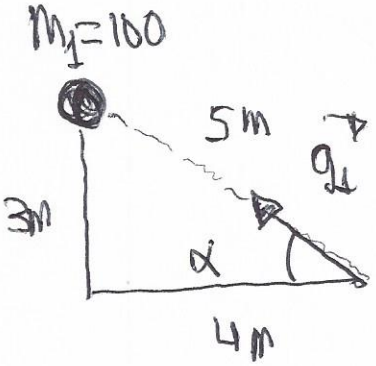
$$|\vec{g}| = 1'79 \cdot 10^{-11}\ N/kg$$

(231)

b)



→ Hallando solamente $\vec{g_1}$ podría escribir $\vec{g_2}$ y componerlo ya que existe igualdad de masas que crean el campo y de distancias.

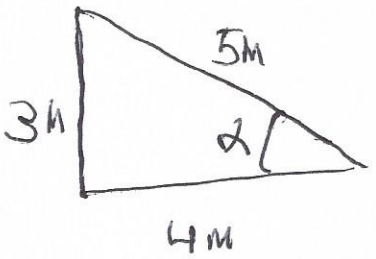


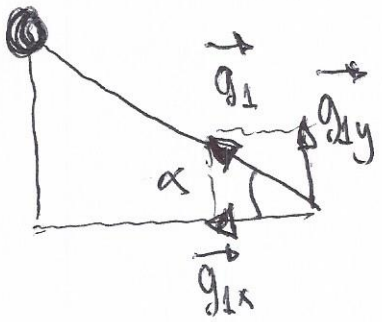
Descompondré g_1

$$|\vec{g_1}| = G \cdot \frac{m_1}{r_1^2} = 6'67 \cdot 10^{-11} \cdot \frac{100}{5^2} = 2'67 \cdot 10^{-10}\ N/s^2.$$

$$\operatorname{sen}\alpha = \frac{3}{5}$$

$$\alpha = \operatorname{arcsen}\frac{3}{5} \Rightarrow \alpha = 36'87°$$





$$\operatorname{sen}\alpha = \frac{g_{1y}}{g_1} \Rightarrow g_{1y} = g_1 \cdot \operatorname{sen} 36'87°$$

$$g_{1y} = 2'67 \cdot 10^{-10} \cdot \frac{3}{5}$$

$$\vec{g_{1y}} = 1'6 \cdot 10^{-10}\vec{j}\ (N/s^2)$$

$$\vec{g_1} = \vec{g_{1x}} + \vec{g_{1y}}$$

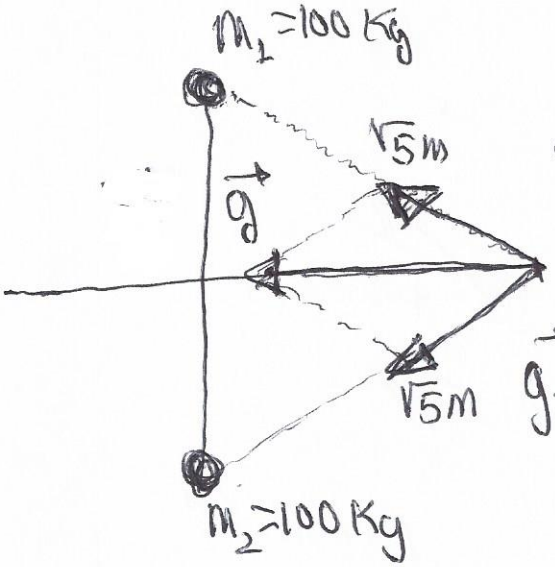
$$\cos\alpha = \frac{g_{1x}}{g_1} \Rightarrow g_{1x} = g_1 \cdot \cos 36'87°$$

$$g_{1x} = 2'67 \cdot 10^{-10} \cdot \frac{4}{5}$$

$$\vec{g_{1x}} = -2'14 \cdot 10^{-10}\vec{i}\ (N/s^2)$$

$$\boxed{\vec{g_1} = -2'14 \cdot 10^{-10}\vec{i} + 1'6 \cdot 10^{-10}\vec{j}\ (N/s^2)}$$

(231) b) Continuación.



$\vec{g_1} = -2'14 \cdot 10^{-10}\vec{i} + 1'6 \cdot 10^{-10}\vec{j}\ \left(M/s^2\right)$

$\vec{g_2} = -2'14 \cdot 10^{-10}\vec{i} - 1'6 \cdot 10^{-10}\vec{j}\ \left(M/s^2\right)$

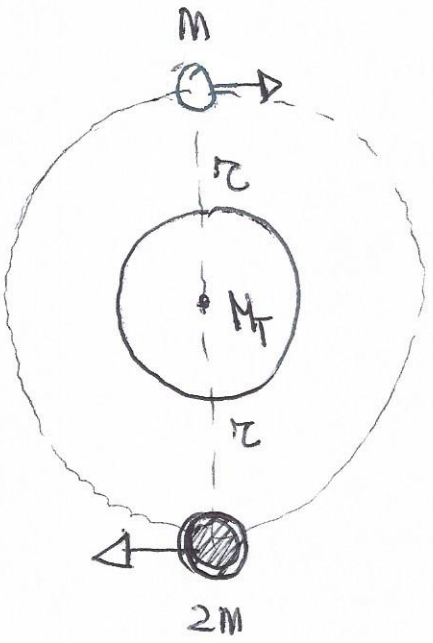
$\vec{g} = \vec{g_1} + \vec{g_2}$ (Principio de Superposición)

$\vec{g} = -2'14 \cdot 10^{-10}\vec{i} + 1'6 \cdot 10^{-10}\vec{j} - 2'14 \cdot 10^{-10}\vec{i} - 1'6 \cdot 10^{-10}\vec{j}$

$$\boxed{\vec{g} = -4'28 \cdot 10^{-10}\,\vec{i}\ \left(M/s^2\right)}$$

$$\boxed{|\vec{g}| = 4'28 \cdot 10^{-10}\ M/s^2}$$

258



a) i) $F_g = F_n$

$$G \cdot \frac{M_T \cdot m}{r^2} = m \cdot a_n$$

$$G \frac{M_T \cdot \cancel{m}}{r^2} = \cancel{m} \cdot \frac{v_0^2}{r}$$

$$v_0 = \sqrt{G \cdot \frac{M_T}{r}} \quad \left(\frac{v_0'}{v_0} = 1\right)$$

La velocidad orbital no depende de la masa del satélite, luego es la misma en ambos casos

ii) Energías totales en las órbitas se refiere a la E_M

$$m \Rightarrow E_m = -G \frac{M_T \cdot m}{2r}$$

$$2m \Rightarrow E_m' = -G \frac{M_T \cdot 2m}{2r}$$

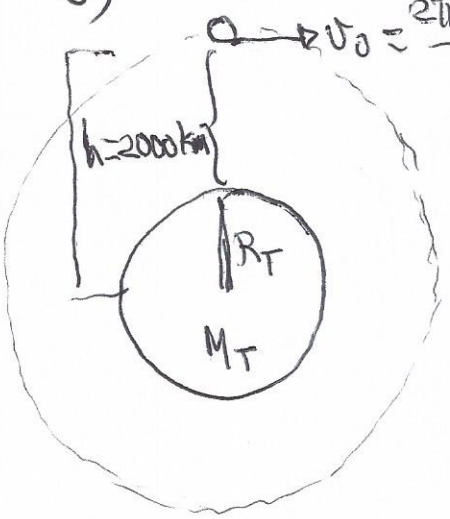
$$\Rightarrow \frac{E_M'}{E_M} = \frac{-G \frac{M_T \cdot 2m}{2r}}{-G \cdot \frac{M_T \cdot m}{2r}}$$

$$\frac{E_M'}{E_M} = 2$$

$$\boxed{E_m' = 2E_m}$$

258

b) i) T?



Calculamos T mediante la condición de orbitación

$F_g = F_n$

$$G \cdot \frac{M_T \cdot m}{(R_T+h)^2} = m \cdot \frac{v_0^2}{(R_T+h)}$$

$$G \cdot \frac{M_T}{(R_T+h)} = \left(\frac{2\pi(R_T+h)}{T}\right)^2$$

$$G \cdot \frac{M_T}{(R_T+h)} = \frac{4\pi^2(R_T+h)^2}{T^2}$$

$$G \cdot M_T \cdot T^2 = 4\pi^2 (R_T+h)^3$$

$$T = \sqrt{\frac{4\pi^2(R_T+h)^3}{G \cdot M_T}}$$

$$T = \sqrt{\frac{4\pi^2 \cdot (6'37 \cdot 10^6 + 2 \cdot 10^6)^3}{6'67 \cdot 10^{-11}, 5'98 \cdot 10^{24}}}$$

$$\boxed{T = 7608'86 \text{ s}}$$

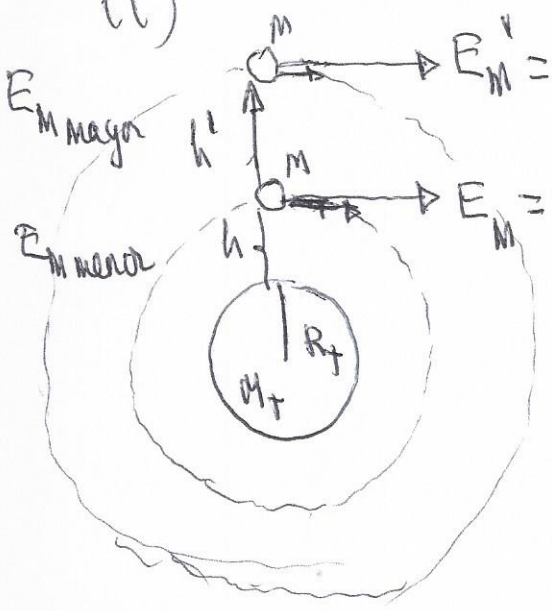
Si ya sabemos T la velocidad orbital podemos calcularla así.

$$v_0 = \frac{2\pi(R_T+h)}{T}$$

$$v_0 = \frac{2\pi(6'37 \cdot 10^6 + 2 \cdot 10^6)}{7608'86}$$

$$v_0 = 6908'21 \text{ m/s}$$

ii)



$$E_M' = -G \frac{M_T \cdot m}{2(R_T+h')} = -6'67 \cdot 10^{-11} \cdot \frac{5'98 \cdot 10^{24} \cdot 5000}{2(6'37 \cdot 10^6 + 5'2 \cdot 10^6)} = -9'48 \cdot 10^{11} \text{ J}$$

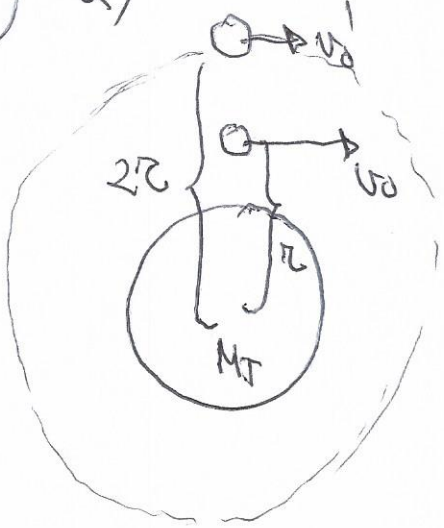
$$E_M = -G \cdot \frac{M_T \cdot m}{2(R_T+h)} = -6'67 \cdot 10^{-11} \cdot \frac{5'98 \cdot 10^{24} \cdot 5000}{2 \cdot (6'37 \cdot 10^6 + 2 \cdot 10^6)} = -1'31 \cdot 10^{12} \text{ J}$$

$$E_m + E = E_M'$$

$$E = E_M' - E_m = -9'48 \cdot 10^{11} \text{ J} - (-1'31 \cdot 10^{12})$$

$E = 3'62 \cdot 10^{11}$ J habrá que aportar

261 a)



Deducción de la velocidad orbital

$$F_g = F_n$$

$$G\frac{M_T \cdot m}{r^2} = m \cdot \frac{v_0^2}{r}$$

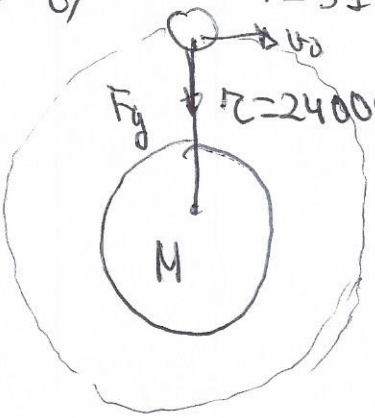
$$\uparrow v_0 = \sqrt{G\frac{M_T}{r}} \downarrow$$

El de menor radio x desplaza a mayor velocidad.

$$\uparrow E_p = -G\frac{M_T \cdot m}{r} \uparrow$$

El de mayor radio tiene mayor E_p

b) i) $T = 31\,h \cdot \frac{3600\,s}{1\,h} = 111600\,s$



Para calcular la masa del planeta M usamos la condición de orbitación sabiendo T del satélite.

$$v_0 = \frac{2\pi r}{T}$$

$$F_g = F_n$$

$$G\cdot\frac{M \cdot m}{r^2} = m \cdot \frac{v_0^2}{r}$$

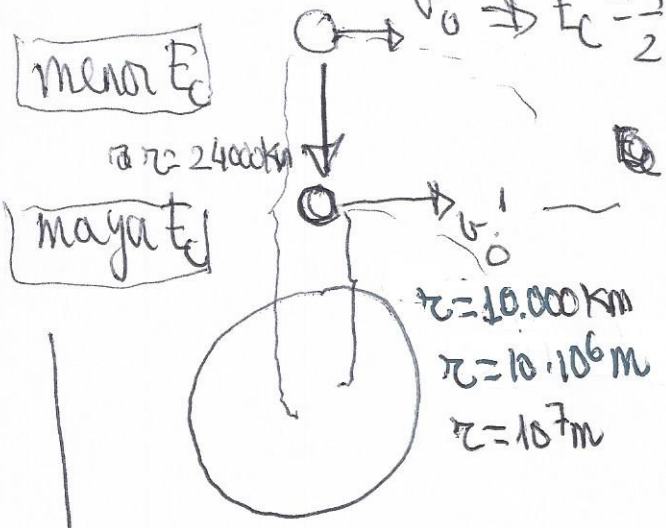
$$G\cdot\frac{M}{r} = \left(\frac{2\pi r}{T}\right)^2$$

$$G\frac{M}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$\boxed{M = \frac{4\pi^2 r^3}{G\cdot T^2}} = \frac{4\pi^2 \cdot (24\cdot 10^7)^3}{6'67\cdot 10^{-11}\cdot (111600)^2} = \frac{5'45\cdot 10^{23}}{0'83} = \boxed{6'57\cdot 10^{23}\,Kg}$$

b)

ii



$\rightarrow v_0 = \sqrt{G\frac{M}{r}}$ (deducida en a)

$v_0 \Rightarrow E_c = \frac{1}{2} m v_0^2 = \frac{1}{2} m \left(\sqrt{G\frac{M}{r}}\right)^2 = \frac{1}{2} G \cdot \frac{M \cdot m}{r}$

$E_c = G \cdot \frac{M \cdot m}{2r} = 6'67 \cdot 10^{-11} \cdot \frac{6'57 \cdot 10^{23} \cdot 500}{2 \cdot 2'4 \cdot 10^7} = 4'56 \cdot 10^8$ J

$E_c' = G \cdot \frac{M \cdot m}{2r'} = 6'67 \cdot 10^{-11} \frac{6'57 \cdot 10^{23} \cdot 500}{2 \cdot 10^7} = 1'09 \cdot 10^9$ J

$\rightarrow$ La E_c habrá experimentado un aumento, se observa que además la nueva velocidad orbital es mayor.

$E_c' - E_c = 1'09 \cdot 10^9 - 4'56 \cdot 10^8 = 6'34 \cdot 10^8$ J

$\Downarrow$

La E_c aumenta en ese tránsito en $6'34 \cdot 10^8$ J.